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HYDRODINAMICS OF LIQUID FILM ON CYLINDRICAL SURFACE

A theoretical study of the film movement of the liquid phase on the surface of a permeable cylinder under the influence of the mass forces of gravity and the swirling gas flow has been carried out. The differential equations of motion were determined for the first time, the exact solutions for the velocity components provided adhesion film on the surface of permeable cylinder and equality of shear stresses at the interface were calculated to determine the thickness of the film and its pressure on the cylindrical surface. The impact of the outflow of the liquid phase on the hydrodynamics of film flow was analyzed. The resulting mathematical model allows taking into account the hydrodynamics of the film during the filtration, separation and heat exchange processes.

Introduction. Analysis method of gas-liquid flows interaction in the phases separation processes shows that the method is promising with swirling flows, which can significantly improve the efficiency of phase separation and heat and mass transfer processes.

Hydrodynamics of film flow on permeable surfaces has significant importance for filtration processes suspensions, removal of the liquid phase in the gas-liquid separator, in heat and mass transfer processes [1]. Suction is also used to manage the boundary layer and influence the stability of laminar motion [2, 3].

It should be noted that the calculations of the parameters affecting the efficiency of devices with film liquid motion in parallel co-current mode or counter-current manner together with swirling gas flow are mainly based on the experimental data.

Mathematical modeling of the processes being investigated allows determining the optimum modes, the ratio between the geometric parameters of cylindrical elements and loads of construction phases.

Main part. Let us consider the steady-state axially symmetric flow of a viscous incompressible liquid on the inner wall of a permeable cylinder under the influence of swirling gas flow (Figure). Z-axis of cylindrical coordinate system is directed right down to the cylinder axis.

Due to axial symmetry $\frac{\partial U}{\partial \varphi} \equiv 0$. We write the Navier - Stokes equations for the velocity and continuity constituents [4]:

$$\rho\left(U_r\frac{\partial U_z}{\partial r} + U\frac{\partial U_z}{\partial z}\right) = \rho g_z - \frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 U_z}{\partial r^2} + \frac{1}{r}\frac{\partial U_z}{\partial r} + \frac{\partial^2 U_z}{\partial z^2}\right); \quad (1)$$

$$\rho\left(U_r\frac{\partial U_\varphi}{\partial r} + \frac{U_\varphi U_r}{r}\right) = \rho g_\varphi + \mu\left(\frac{\partial^2 U_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial U_\varphi}{\partial r} - \frac{U_\varphi}{r^2}\right); \quad (2)$$

$$\rho\left(U_r\frac{\partial U_r}{\partial r} - \frac{U_\varphi^2}{r}\right) = \rho g_r - \frac{\partial P}{\partial r} + \mu\left(\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r}\frac{\partial U_r}{\partial r} - \frac{U_r}{r^2}\right); \quad (3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rU_r\right) + \frac{\partial U_z}{\partial z} = 0. \quad (4)$$

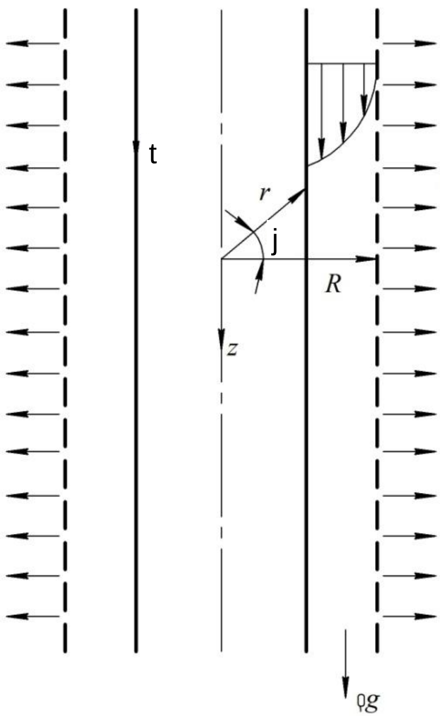

Fig 1. Two- phase film fluid motion on permeable surface.

Outflow rate of the liquid phase U_0 at some elementary cylinder Δz by length, is assumed to be constant. Volumetric flow rate of a non-compressible fluid through a cylindrical surface of equal length will be equal: $2\pi r U_r \Delta z = 2\pi R U_0 \Delta z$. Hence we calculate a radial velocity in the liquid film $U_r = \dfrac{U_0 R}{r}$. Then we obtain $\dfrac{\partial U_z}{\partial z} = 0$ and $U_z = U_z(r)$. from the continuity equation and assume $\psi = \dfrac{\partial P}{\partial z} = \text{const.}$

Equations (1) - (4) are converted to the form:

$$\frac{d^2U_z}{dr^2} - \frac{1}{r}\left(\frac{U_0R}{\nu} - 1\right)\frac{dU_z}{dr} = -\frac{\rho g - \psi}{\mu};$$

$$\frac{d^2U_\varphi}{dr^2} - \frac{1}{r}\left(\frac{U_0R}{\nu} - 1\right)\frac{dU_\varphi}{dr} - \frac{1}{r^2}\left(\frac{U_0R}{\nu} + 1\right)U_\varphi = 0;$$

$$\frac{\partial P}{\partial r} = \rho\left(\frac{U_\varphi^2}{r} + \frac{U_0^2R^2}{r^3}\right)$$

As a result, we obtain a system of ordinary differential equations. This means that the solution $U = U(r)$ will be self-similar. Let us transfer to the dimensionless coordinate $\xi = r/R$, and denote $\alpha = \dfrac{U_0R}{\nu}$, and obtain

$$\frac{d^2U_z}{d\xi^2} - \frac{(\alpha - 1)}{\xi}\frac{dU_z}{d\xi} = -\frac{\rho g - \psi}{\mu}R^2; \tag{5}$$

$$\frac{d^2U_\varphi}{d\xi^2} - \frac{(\alpha - 1)}{\xi}\frac{dU_\varphi}{d\xi} - \frac{(\alpha + 1)}{\xi^2}U_\varphi = 0; \tag{6}$$

$$\frac{\partial P}{\partial r} = \rho\left(\frac{U_\varphi^2}{r} + \frac{U_0^2R^2}{r^3}\right). \tag{7}$$

For boundary conditions we use the values of the fluid velocity constituents on the cylindrical surface and the components of the tensor of shear stresses

$$\tau_z = -\mu\frac{\partial U_z}{\partial r};\quad \tau_\varphi = -\mu\left(\frac{\partial U_\varphi}{\partial r} - \frac{U_\varphi}{r}\right) \tag{8}$$

at the phase separation boundary.

Particular solutions of equations (5) - (7) are in the form r^k and obtain general solutions:

$$U_z = c_1 + c_2\xi^\alpha + \frac{\rho g - \psi}{2\mu(\alpha - 1)}\xi^2; \tag{9}$$

$$U_\varphi = \frac{c_3}{\xi} + c_4\xi^{\alpha+1}. \tag{10}$$

For the boundary conditions we assume adhesion condition on the wall, and shear stresses equality at the phase separation boundary:

$$U_z\big|_{\xi=1} = U_\varphi\big|_{\xi=1} = 0;\quad \tau_z = -\frac{\mu}{R}\frac{dU_z}{d\xi}\bigg|_{\xi=1-\delta}; \tag{11}$$

$$\tau_\varphi = -\mu\left(\frac{\partial U_\varphi}{R\partial\xi} - \frac{U_\varphi}{R\xi}\right)\bigg|_{\xi=1-\delta}. \tag{12}$$

Due to the equilibrium of forces acting on the gas flow

$$\pi(R - \delta)^2\Delta P = 2\pi(R - \delta)\tau'_z l,$$

obtain:

$$\psi = \frac{\Delta P}{l} = \frac{2\tau'_z}{R(1 - \delta)} = -\frac{2\tau_z}{R(1 - \delta)}$$

Considering the boundary conditions (11) - (12), we obtain the velocity distribution in the liquid film:

$$U_z = \left[\frac{\tau_z R}{\mu(\alpha - 2)(1 - \delta)^{\alpha-1}} + \frac{\rho g R^2}{\mu\alpha(\alpha - 2)(1 - \delta)^{\alpha-2}}\right] \times (1 - \xi^\alpha) -$$

$$-\left[\frac{\rho g R^2}{2\mu\alpha(\alpha - 2)} + \frac{\tau_z R}{\mu(\alpha - 2)(1 - \delta)}\right](1 - \xi^2); \tag{13}$$

$$U_\varphi = \frac{R\tau_\varphi(1 - \delta)^2}{\mu\alpha(1 - \delta)^{\alpha+2} + 2\mu}\left(\frac{1}{\xi} - \xi^{\alpha+1}\right). \tag{14}$$

Integrating the obtained dependencies (13) - (14), we calculate that the volume flow rate of the fluid phase per perimeter unit, the average value of the tangential component of the velocity of the film and the pressure drop in the radial direction:

$$q = R\int_{1-\delta}^{1} U_z\xi\,d\xi; \tag{15}$$

$$\bar{U}_\varphi = \frac{1}{\delta}\int_{1-\delta}^{1} U_\varphi\,d\xi; \tag{16}$$

$$\Delta P_r = \rho\int_{1-\delta}^{1}\left(\frac{U_\varphi^2}{r} + \frac{U_0^2R^2}{r^3}\right)d\xi. \tag{17}$$

Development of dependencies (15) in series to the fourth degree inclusive has the form:

$$q = \frac{\tau_z\delta^2}{\mu}\left[\frac{1}{2} + \frac{1}{3}\frac{U_0\delta}{\nu} + \frac{1}{8}\left(\frac{U_0^2\delta^2}{\nu^2} + \frac{\delta^2}{R^2}\right) + \ldots\right] +$$

$$+\frac{\rho g\delta^3}{\mu}\left[\frac{1}{3} + \frac{1}{24}\left(3\frac{U_0\delta}{\nu} - 8\frac{\delta}{R}\right) + \ldots\right]. \tag{18}$$

This development shows that the change in the hydrodynamic characteristics due to the outflow in a liquid phase occurs when the dimensionless complex Reynolds $\frac{U_0\delta}{\nu}$ can be compared with the unit. Change of the current volumetric flow along the length is described by the equation

$$\frac{dq}{dz} = -U_0. \qquad (19)$$

This development shows that the change in the hydrodynamic characteristics due to the outflow in a liquid phase occurs when the dimensionless complex Reynolds $\frac{U_0\delta}{\nu}$ can be compared with the unit.

Conclusion. A mathematical model was developed to determine the hydrodynamic characteristics of film flow under the action of a swirling gas flow, taking into account the outflow of the liquid phase. This model takes into account the hydrodynamics of the film in the investigation of filtration processes, separation and heat and mass transfer.

For the first time, the theoretical research was carried out to demonstrate the method gas-film flow filtration, to develop a method for calculating the separation of multiphase flow in the fields of mass forces, effective separation of gas-liquid flows with the removal of the liquid phase, and heat and mass transfer processes.

Notations: c_1, c_2, c_3, c_4 - constant coefficients; g - acceleration of free fall, m/s^2; l - length of the cylindrical permeable element, m; P - pressure, Pa; ΔP - the pressure drop, Pa / m; q – specific volumetric flow rate of the liquid phase m^3 / $(m{\cdot}s)$; r - radial distance in a cylindrical coordinate system, m; R - radius of the permeability of the cylindrical element, m; $\tilde{r} = r / R$ - dimensionless radial coordinate; U_z, U_φ, U_r - axial, radial and tangential component of the velocity of the fluid, respectively, m / s; U_0 - outflow rate of fluid through the permeable surface, m / s; ν - the kinematic viscosity, m^2 / s; z - axial coordinate of the cylindrical coordinate system, s/m^2; α - coefficient, m; δ - thickness of the liquid film, m; $\tilde{\delta} = \delta / R$ = dimensionless thickness of the liquid film, m; μ - coefficient of dynamic viscosity, Н×/м^2; π = 3.14159…; ρ – density, kg/m^3; τ - shear stress, N/m^2; φ - angle in a cylindrical coordinate system; ψ - pressure drop, Pa / m.

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