MULTITHRESHOLD MAJORITY DECODING OF LDPC-CODES

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Absract. *The article deals with the majority decoding LDPC-codes - three-dimensional linear iterative codes. The possibility of correcting different types of multiple errors is analyzed. The expediency of using multithreshold majority decoding with codes, belonging to this class is presented.*

Keywords: multithreshold majority decoder, multidimensional iterative code, LDPC-code, parity, error correction

DEKODOWANIE KODÓW LDPC O WIELOPROGOWEJ WAŻNOŚCI

Streszczenie. *W artykule zostały przeanalizowane aspekty zastosowania większościowej metody dekodowania kodów LDPC – trójwymiarowych liniowych kodów korekcyjnych. Została przeanalizowana możliwość korekcji różnych typów zwielokrotnionych błędów. Udowodniono zasadność wykorzystania wieloprogowego dekodowania większościowego z kodami danej klasy.*

Słowa kluczowe: wieloprogowy dekoder większościowy, wielowymiarowy kod iteracyjny, parzystość, korekcja błędów

Introduction

The reliability of data storage and transmission of binary data is one of the major problems in today's information society. Increasing the density of elements integration in information storage and transmission systems leads to an increased probability of the appearance of higher multiplicity errors. Using redundant codes can solve the described problem. There are many different codes with high-correcting capabilities (for example BCH codes, Reed-Solomon codes, low-density parity-check code (LDPC) and others). But the decoder plays the crucial role in the error correcting process. So, the purpose of this work is to study the multithreshold majority decoding of LDPC-codes.

1. Principal part

Two-dimensional iterative codes, which are widely used in practice and more commonly known as HV-codes, are the simplest example of the application of methods of known codes combination for the construction of new codes and represent a direct multiplication of simple code parity. The progressive development of redundant encoding in this direction led to the appearance of two-dimensional linear iterative codes with diagonal checks [3], and also their three-dimensional versions [1]. Three-dimensional iterative codes can be attributed to LDPC-codes due to the low-density units in the generator matrix (in rows of check matrix no more than $\sqrt[3]{k}$ units).

The principle of check symbols formation for codes of this class demonstrated by a linear three-dimensional iterative code with double incorporated diagonal checks (five linearly independent parities) with $k = 64$ bits is shown in Fig. 1 (1 – data bits, 2 – horizontal parities, 3 – vertical parities, 4, 5 – respectively the first and second diagonal parities combined, 6 – z-parities, 7 – check sum).

Check bits $R_{1\text{–}80}$ with $k = 64$ in accordance with fig.1 can be calculated using the following relationships:

$$\begin{aligned} R_1 &= X_1 \oplus X_2 \oplus X_3 \oplus X_4, \\ R_2 &= X_5 \oplus X_6 \oplus X_7 \oplus X_8, \\ &\vdots \\ R_5 &= X_1 \oplus X_5 \oplus X_9 \oplus X_{13}, \\ &\vdots \\ R_{14} &= X_3 \oplus X_8 \oplus X_9 \oplus X_{14}, \\ &\vdots \\ R_{16} &= X_1 \oplus X_6 \oplus X_{11} \oplus X_{16}, \end{aligned} \tag{1a}$$

$$\begin{aligned} R_{17} &= X_{17} \oplus X_{18} \oplus X_{19} \oplus X_{20}, \\ &\vdots \\ R_{37} &= X_{33} \oplus X_{37} \oplus X_{41} \oplus X_{45}, \\ &\vdots \\ R_{60} &= X_{52} \oplus X_{55} \oplus X_{58} \oplus X_{61}, \\ &\vdots \\ R_{65} &= X_1 \oplus X_{17} \oplus X_{33} \oplus X_{49}, \\ &\vdots \\ R_{80} &= X_{16} \oplus X_{32} \oplus X_{48} \oplus X_{64}. \end{aligned} \tag{1b}$$

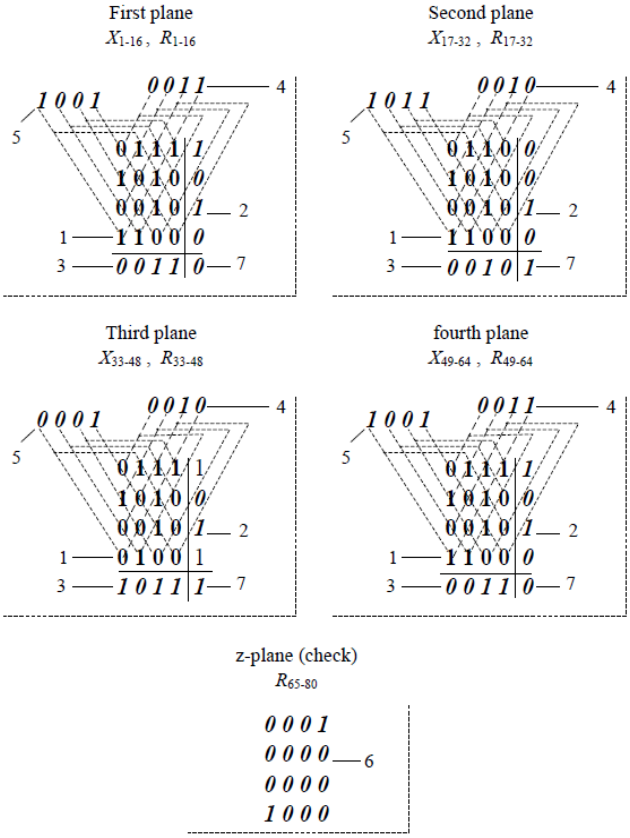

Fig. 1. The principle of check symbols formation by three-dimensional line iterative code with double incorporated diagonal checks

Note, for the error correction by multithreshold majority decoder it is necessary to exclude the checksum in the planes with the information bits, because these are linearly dependent checks and can be obtained by the sum of all horizontal and vertical parities in the appropriate plane.

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In the check matrix of multidimensional linear iterative codes we suggest using up to 9 different linearly independent parities, namely:

- horizontal parities in the plane (Fig. 1);
- vertical parities in the plane (Fig. 1);
- two type incorporated diagonal parities in the plane (Fig. 1);
- z-parities between the planes (Fig. 1);
- z-parities with horizontal shift between planes;
- z-parities with vertical shift;
- two types of z-parities with a diagonally shift between the planes.

Using a larger number of parities can cause the loss of low-density codes properties.

Three-dimensional linear iterative codes are characterized by a minimum code distance equal to

$$d_{\min} = p + 1, \tag{2}$$

where p is the number of linearly independent parities, so any two code words will be different at least in p+1 positions. Therefore, when using such a code (with odd number of linearly independent parity) all the errors can be corrected, providing their multiplicity does not exceed $(p-1)/2$, and all the errors which do not exceed the multiplicity $(p+1)/2$ can be found.

In practice, for instance in satellite communication systems, LDPC-codes are used for informational sequences of hundreds or thousands bits length. However, according to the coding theory, the main parameters of codes are redundancy (r) and code rate ($R = k / (k + r)$) [3]. These parameters for three-dimensional linear iterative codes with different number of linearly independent parities for various k lengths (from 64 to 4096 bits) are presented in table1.

As shown in table 1, the rate code increases with the increasing in the information sequence length.

However, the key role in the use of redundant codes belongs to the decoding process. A majority decoder - decoding on the principle of "majority" - is one of the fastest. The idea of multithreshold majority decoding (MTD) [2] has also been actively developed lately.

The basis of the MPD is the iterative decoding. During the basic step of decoding all k information symbols of messages can be verified many times and in any order. Some bits may be falsely inverted. Some of these errors will be corrected during the following attempts to decode the same bits (i.e., on subsequent iterations). The main property of the MTD is constant convergence of its solutions to optimal decoder solution (with the maximum of the probability).

It is recommended for the first iterations of decoding to use the maximum threshold value with following approaching to the minimum threshold value for subsequent iterations that significantly improves the overall efficiency of the decoder. While using multidimensional linear iterative code the maximum threshold should be considered equal to the number of linearly independent parities $(T_{\max} = p)$, and the minimum value determined by the formula $T_{\min} = round(T_{\max} / 2)$.

To analyze the expediency of using multithreshold decoders in the process of correcting both independent and module multiple errors a program model was developed. It simulates the processes of encoding/decoding, appearance and correction of errors.

All researches were carried out using multidimensional linear iterative code with five linearly independent parities. The decoding process is carried out in 3 iterations with the threshold values $T_1 = 5, T_2 \geq 4, T_3 \geq 3.$

The simulation results of the decoding process of information sequences with triple and quadruple independent errors and length k from 64 to 512 bits are presented in Fig. 2.

The simulation results of the decoding process of information sequences with length k from 64 to 512 bits and module errors are presented in table 2. Note that for the effective correction of module error it is necessary to use multidimensional linear iterative codes with a larger number of checks between the planes than the number of checks in the plane. When using a large number of checks in the plane for an effective correction of module error it is necessary to use an interleaver.

Table 1. Parameters of low-density three-dimensional iterative code

The length of an information sequence, k	Five linearly independent parities		Seven linearly independent parities		Nine linearly independent parities	
	redundancy, *r*	code rate, *k* / (*k* + *r*)	redundancy, *r*	code rate, *k* / (*k* + *r*)	redundancy, *r*	code rate, *k* / (*k* + *r*)
64	80	0,44	112	0,36	144	0,31
512	320	0,62	448	0,53	576	0,47
4096	1280	0,76	1792	0,70	2304	0,64

Table 2. The results of module error correction

The length of information sequence	Code matrix	Quantity of corrected errors with different multiplicity, %							
		3	4	5	6	7	8	9	10
64	4x4x4	100	100	100	67.63	78.99	78.10	56.62	62.22
128	4x4x8	100	100	47.46	47.23	47.01	46.78	46.55	46.32
128	4x8x4	100	100	100	100	100	100	75.81	63.56
128	8x4x4	100	100	100	62.95	75.60	75.10	50.81	60.32
256	8x8x4	100	100	100	100	100	100	71.82	57.63
512	8x8x8	100	100	100	100	100	100	100	46.42

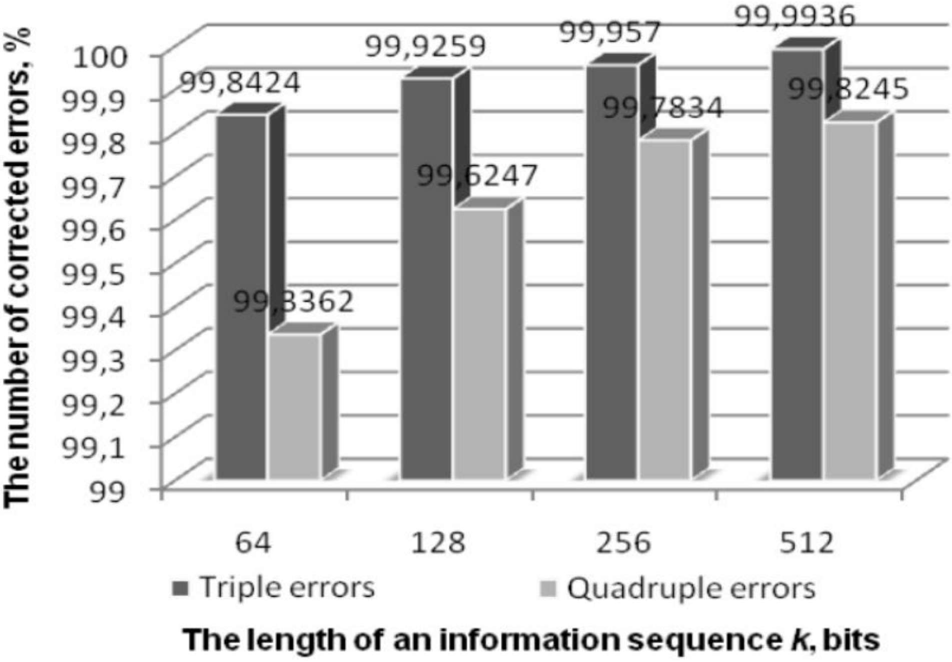

Fig. 2. Diagram of triple and quadruple corrected errors

2. Conclusions

So, the research results of the error correction with the help of three-dimensional linear iterative code (on the example of the code with five linearly independent parities, three of them between the planes) and multithreshold decoder can be represented by the following conclusions.

1. Multithreshold decoder allows to correct multiple errors of module type whose multiplicity is no greater than the number of columns in the code plane (a linear three-dimensional iterative code was used).

2. With increasing length of the information sequence and growing code rate of three-dimensional linear iterative code we can observe the increase in the quantity of triple and quadruple errors corrected. For example, using a code with five linearly independent parity and information sequence 512 bits, 99.99% triple errors and 99.92% quadruple errors can be corrected.

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