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**CALCULATION OF FLOTATOR'S AERATOR FOR SEPARATION  
OF GROUND PLASTICS**

**Abstract.** When creating a new equipment for flotation separation of a mixture of ground secondary plastics, it is necessary to take into account the peculiarities of their properties due to the nature of polymers. So, in comparison with mineral materials, the surface of the ground plastics is less developed, and the particle size is larger. These factors reduce the strength and stability of the “bubble – particle” complex. Turbulent pulsations in the working volume of a flotator for separating a mixture of ground plastics adversely affect the flotation efficiency, since the resulting inertial forces destroy the “bubble – particle” complex. Therefore, the turbulent motion of dispersed phases in the working volume of the apparatus is not desirable. The purpose of this work was to determine the structural and technological parameters of a pneumatic aerator, which will ensure the absence of turbulent motion of bubbles and particles. For this purpose, a method of engineering calculation of the structural and technological parameters of the flotator's pneumatic aerator was created. The pneumatic aerator's design was chosen as an Archimedes' spiral. The engineering calculation method includes the following stages. From the condition of the bubbles' laminar motion, the maximum allowable relative velocity of the spherical bubble and its diameter are calculated. Knowing the optimal gas content in the column, the relative velocity of the bubbles' constrained floating up is calculated. The maximum equivalent particle diameter that the bubble can rise into the foam layer is determined. The spiral pitch (and the perforation pitch) is determined. The aerator's tube length is calculated. The allowable consumption is found. Based on the calculated

parameters, a laboratory installation of a column flotator with pneumatic liquid aeration was created. It studied the bubbles' dispersion formed during the liquid aeration. The values of the calculated and experimental average gas bubble diameter were compared.

**Key words:** Bubble, particle, aerator, diameter, motion, flotation, hole.

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## **ҰНТАҚТАЛҒАН ПЛАСТМАССАНЫ БӨЛУГЕ АРНАЛҒАН ФЛОТАЦИЯЛЫҚ АППАРАТТЫ АЭРАТОРДЫ ЕСЕПТЕУ**

**Аннотация.** Ұнтақталған екінші реттік пластмасса қоспасын флотациялық бөлуге арналған жаңа жабдықты жасау кезінде полимерлердің табиғатына байланысты қасиеттерінің ерекшеліктерін ескеру қажет. Сонымен, минералды материалдармен салыстырғанда ұсақталған пластмасса беті онша дамымаған, ал бөлшектер мөлшері үлкен. Бұл факторлар көпіршік-бөлшек кешенінің беріктігі мен тұрақтылығын төмендетеді. Ұнтақталған пластмасса қоспасын бөлуге арналған флотациялық аппараттың жұмыс көлеміндегі турбулентті пульсациялар флотация тиімділігіне кері әсер етеді, өйткені пайда болатын инерциялық күштер «көпіршік - бөлшек» кешенін бұзады. Демек, аппараттың жұмыс көлеміндегі дисперсті фазалардың турбулентті қозғалысы құптарлық емес. Бұл жұмыстың мақсаты көпіршіктер мен бөлшектердің турбулентті қозғалысының болмауын қамтамасыз ететін пневматикалық қопсытқыштың құрылымы мен технологиялық параметрлерін анықтау болды. Ол үшін флотациялық аппараттың пневматикалық қопсытқышының жобалық және технологиялық параметрлерін инженерлік есептеу әдістемесі жасалды. Пневматикалық қопсытқыштың дизайны Архимед спиралы түрінде таңдалды. Инженерлік есептеу әдістемесі келесі кезеңдерді қамтиды. Көпіршіктердің ламинарлы қозғалысының шартынан сфералық көпіршіктің рұқсат етілген шекті салыстырмалы жылдамдығын және оның диаметрін есептеу. Бағандағы оңтайлы газ құрамын біле отырып, көпіршіктердің шектелген көтерілуінің салыстырмалы жылдамдығы есептеледі. Көпіршікті көбік қабатына көтере алатын бөлшектердің максималды эквивалентті диаметрі анықталады.

Аэратор саңылауларының мөлшерін анықталады. Аэратор түтігінің сыртқы диаметрін біле отырып, аэратор деңгейінен төмен бөлшектердің еркін өту шартын қанағаттандыратын қопсытқыш саңылауларының мүмкін болатын максималды мәні алынады. Одан кейін спиральдың қадамын анықтау (және перфорацияның қадамы) жүзеге асырылады. Аэратор түтігінің ұзындығын есептеліп, ағынның рұқсат етілген жылдамдығын табамыз. Есептелген параметрлер бойынша пневматикалық сұйықтық аэрациясы бар бағаналы флотация аппаратының зертханалық қондырғысы құрылды. Нәтижесінде сұйықтықтың аэрациясы кезінде пайда болған көпіршіктердің дисперсиясы зерттелді. Газ көпіршігінің есептелген және тәжірибелік орташа диаметрінің мәндері салыстырылды.

**Түйін сөздер:** көпіршік, бөлшек, қопсытқыш, диаметр, қозғалыс, флотация, тесік.

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## **РАСЧЕТ АЭРАТОРА ФЛОТАЦИОННОГО АППАРАТА ДЛЯ РАЗДЕЛЕНИЯ ИЗМЕЛЬЧЕННЫХ ПЛАСТМАСС**

**Аннотация.** При создании нового оборудования для флотационного разделения смеси измельченных вторичных пластмасс следует учитывать особенности их свойств, обусловленные природой полимеров. Так, по сравнению с минеральными материалами, поверхность измельченных пластмасс менее развита, а размер частиц больше. Эти факторы снижают прочность и стабильность комплекса «пузырек – частица». Турбулентные пульсации в рабочем объеме флотационного аппарата для разделения смеси измельченных пластмасс отрицательно влияют на эффективность флотации, поскольку возникающие инерционные силы разрушают комплекс «пузырек – частица». Следовательно, турбулентное движение дисперсных фаз в рабочем объеме аппарата нежелательно. Целью настоящей работы являлось определение конструктивных и технологических параметров пневматического аэратора, которые обеспечат отсутствие турбулентного движения пузырьков и частиц. Для этого была создана методика инженерного расчета конструктивных и технологических параметров пневматического

аэратора флотационного аппарата. Конструкция пневматического аэратора была выбрана в виде спирали Архимеда. Методика инженерного расчета включает в себя следующие этапы. Из условия ламинарного движения пузырьков вычисляют максимально допустимую относительную скорость сферического пузырька и его диаметр. Зная оптимальное газосодержание в колонне, рассчитывают относительную скорость стесненного всплытия пузырьков. Определяют максимальный эквивалентный диаметр частицы, которую пузырек сможет поднять в пенный слой. Определяют размер отверстий аэратора. Зная наружный диаметр трубки аэратора, получают максимально возможное значение числа отверстий аэратора, удовлетворяющих условию свободного прохождения частиц ниже уровня аэратора. Определяют шаг спирали (и шаг перфорации). Рассчитывают длину трубки аэратора. Находят допустимый расход. По рассчитанным параметрам была создана лабораторная установка колонного флотационного аппарата с пневматической аэрацией жидкости. В ней была исследована дисперсность пузырьков, образующихся при аэрации жидкости. Были сопоставлены значения расчетного и экспериментального среднего диаметра пузырька газа.

**Ключевые слова:** пузырек, частица, аэратор, диаметр, движение, флотация, отверстие.

**Introduction.** Since the flotation process arose long before the wide distribution and use of plastics and was used for separation of mineral ores, the majority of scientific research in the field of flotation was performed for these purposes. Consequently, during the flotation of plastics, it is necessary to take into account their important features due to the nature of polymers.

**Literature data analysis and problem definition.** Before the flotation, at the stage of preparation, all materials are subjected to grinding. Unlike mineral ores, which are grinded mainly in hammer, jaw crushers, rod, ball or rotary centrifugal mills, where impact and abrasion loads prevail, plastics are grinded more often in shredder or knife machines, where cutting loads prevail. The ground plastics surface is less developed due to the plastic, viscous properties of polymers, and also due to the grinding method. For the same reasons, the size of the plastic particles entering the flotation is larger than that of mineral materials. All these features of flotation of plastic particles reduce the strength and stability of the “bubble – particle” complex.

The bubbles and particles’ motion mode is a significant factor affecting the possibility of the “bubble – particle” flotation complex formation, the flotation intensity and the process’ energy consumption. The probability of the bubble and particle impact, as well as the flotation complex formation, depends on

the relative velocity of their motion, duration of contact, and inertial forces (Volnenko A.A et.al, 2020).

Since the bubble density is three orders of magnitude lower than that of the liquid and particles, therefore, gas bubbles are the most dynamic dispersed phase in the flotator's working volume. Therefore, special requirements are imposed on the bubbles' motion mode in the flotator's working volume.

With a certain critical size of the gas bubble and the value of the Reynolds number, the dynamic effect of the liquid medium increases and the surface tension force influence on the bubble shape decreases. At the same time, the bubble shape's pulsation effects due to the internal gas circulation are manifested. Deformation of the bubble and deviation of its shape from spherical occur. The instability of the bubble shape leads to changes in velocity and violations in vertical trajectory of its floating up, which takes the shape of a flattened spiral, Fig. 1.

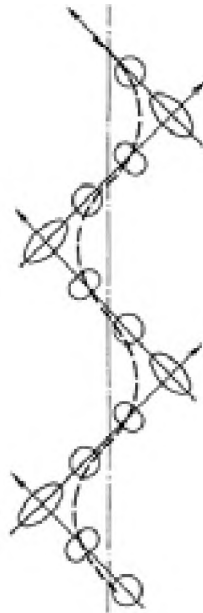


Fig. 1. – Trajectory of a large bubble floating up

For the flotation of large hard-to-remove particles, such a trajectory of motion is undesirable, since there is a separation of the particle from the bubble under the influence of turbulence.

In this case, two variants of separation are likely. The first variant is to destroy the “bubble – particle” complex under the influence of forces arising due to the turbulent pulsations. These forces appear due to the pressure difference on

the opposite sides of the complex and tangential stresses (as a consequence of the velocity gradient). The second variant of separation occurs under the action of centrifugal forces, when the “bubble – particle” complex moves along a curvilinear trajectory.

More precisely, the bubble shape can be determined using the values of the Reynolds, Morton, Eötvös (Bond), Weber numbers (Ziegenhein T. et.al, 2017). However, in the literature there are data that the bubble sphericity can be preserved up to the Reynolds number equal to 300.

$$\text{Re} < 300. \quad (1)$$

Re – relative Reynolds number, which is determined as

$$\text{Re} = \frac{\rho_1 d_2 |\vec{u}_2 - \vec{u}_1|}{\mu_1}. \quad (2)$$

where  $d_2$  – the dispersed phase diameter, m;

$\rho_1$  – the dispersion phase density (medium density),  $\text{kg/m}^3$ ;

$\vec{u}_2$  – the dispersed phase velocity, m/s;

$\vec{u}_1$  – the dispersion phase velocity, m/s;

$\mu_1$  – the dynamic liquid viscosity, Pa·s.

The velocity difference between the dispersed and dispersion phases is the relative velocity [4].

We believe that the flotation separation of the mixture of ground plastics should be carried out while the gas bubbles are spherical in shape, i.e., if condition (1) is fulfilled.

**Objective and tasks of the research.** The objective of this work is to determine the structural and technological parameters of the pneumatic aerator, which will ensure the absence of the turbulent bubbles and particles motion.

To achieve this objective, it is necessary to solve the following tasks:

- determination of the maximum allowable relative velocity of the spherical bubble and its diameter;
- taking into account the optimal gas content in the column, calculate the relative velocity of the constrained floating up of bubbles;
- determination of the maximum equivalent particle diameter that the bubble can rise into the foam layer;
- determination of the aerator's holes size and their number.

Research methods. In this work, it was used the method of direct photographing the air bubbles in the volume of liquid and on its surface together with a scale ruler.

**Method of engineering calculation of the structural and technological parameters of the flotator's pneumatic aerator.** An intense interaction between the liquid and the bubbles, their destruction and coalescence occur in a multiphase flow (Wang T. et.al, 2007). To find the instantaneous velocity and location of the dispersed phase, for each of its position, the liquid's velocity field must be known (the liquid's instantaneous velocity at all its points) (Kuan B. et.al, 2007). An exact mathematical description of such complex flows was not found.

In complex multiphase flows, the velocity field is unknown and difficult to be measured. Nevertheless, under certain conditions, the task can be significantly simplified to determine the velocity of the dispersed phase as a first approximation (Ziegenhein T. et.al, 2017). Various scientists in their works (Liu L. et.al, 2016; Zhou R. et.al, 2017) draw conclusions that the resistance force has a much more significant effect in contrast to other forces acting on the dispersed phase (Shang Z. et.al, 2015), (the force created by the virtual mass effect arising from the replacement of the liquid volume by the moving particle volume (Kuan B. et.al, 2007), the lifting force due to the relative velocity gradient over the dispersed phase surface (Crowe C.T, 2011), the force arising due to the pressure gradient (Kuan B. et.al, 2007), the turbulent dispersion force due to the motion of dispersion turbulent vortices, the thermal diffusion force (Talbot L. et.al, 1980) (is taken into account for particles less than 1 mm), the photodiffusion force, the Brownian force (Li A. et.al, 1992), etc.).

Suppose that in the flotator's working volume the velocity field is uniform, stationary, there are no turbulent vortices. Consequently, the lifting force, the turbulent dispersion force and the force arising from the pressure gradient can be neglected. We believe that the dispersed phase moves uniformly, since the relaxation time (during which it reaches a constant velocity) is negligible.

Then the moving force due to the difference in the densities of the dispersion (the medium density) and the dispersed phases will be balanced by the resistance force acting from the liquid. This equilibrium condition in the massless recording form will be:

$$\frac{\vec{u}_2 - \vec{u}_1}{\tau_2} = \vec{g} \frac{\rho_1 - \rho_2}{\rho_2}, \quad (3)$$

where  $\tau_2$  – the dispersion phase's relaxation time, s;

$\vec{g}$  – the free fall acceleration, 9.81 m/s<sup>2</sup>;

$\rho_2$  – the dispersed phase density, kg/m<sup>3</sup>.

The relaxation time is a measure of inertia and denotes the time scale of the relative velocity approaching a stable constant velocity [3, 6, 11]:

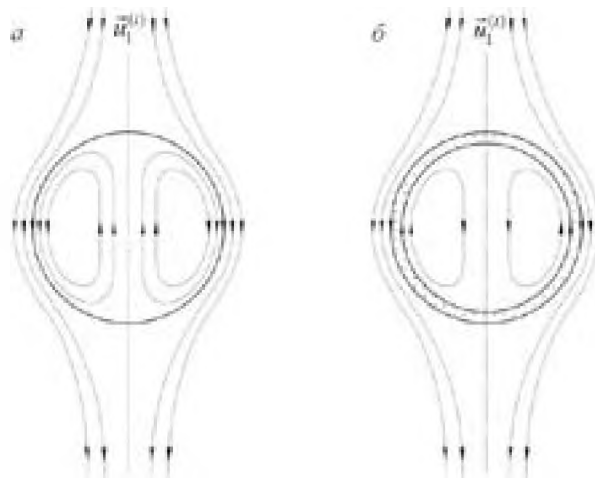
$$\tau_2 = \frac{\rho_2 d_2^2}{18\mu_1} \frac{24}{C_D \text{Re}} = \frac{4\rho_2 d_2^2}{3\mu_1 \text{Re} C_D}, \quad (4)$$

where  $C_D$  – the coefficient of resistance.

In accordance with different motion types of dispersed particles in the liquid flow, the coefficient of resistance can be calculated for different regions of the Reynolds number. There are various models for determining the coefficient of resistance, for example, Schiller-Neumann, the Morsi model, the Stokes law, the Heider model, the Ishiya model, the Grace model, Tomiyama, the universal models, the models for determining the coefficient of resistance of solid particles (Huilin L. et al, 2003). Other models are also known (Rodrigue D. et al, 1996).

When comparing the experimental data obtained during the floating up of gas bubbles in liquids without special measures for their purification, with the experimental data that were obtained with thorough purification of aqueous solutions, it was found (first by A.N. Frumkin and V.G. Levich (Levich V.G 1959), later – by others (Polyanin A.D. et al, 2002)) that even very small amounts of surfactants lead to the fact that the actual velocity of the bubbles' floating up and the coefficient of resistance are close to the values obtained for the solid spherical particle.

In the bubble as it moves in the purified liquid flow, internal gas circulation occurs as shown in Fig. 2, a. The circulation direction at the phase boundary coincides with the liquid flow at the same surface.



a – in the flow of the purified liquid; b – in the flow with surfactants

Fig. 2. – Trajectory of the liquid and gas flow lines during the bubble floating up.



While the bubble moves in the flow with surfactants leads to the “hardening” of the bubble surface, suppressing the internal gas circulation (Fig. 2, б). This is explained by the fact that surfactants dissolved in the liquid accumulate on the bubble surface and move along it under the influence of the liquid flow. In this case, the surface tension gradient arises, forcing the surfactant to move in the opposite direction. Therefore, in the presence of surfactant, the bubble motion velocity is reduced, and its coefficient of resistance may increase.

The same effect was discovered by A. V. Gorodetskaya both for large bubbles (Polyanin A.D. et.al, 2002) and in the region of intermediate Reynolds numbers, i.e., at  $2 < Re < 300$ . The coefficient of resistance and motion velocity of the spherical bubbles affected by the presence of surfactants in the liquid should be calculated using the equations obtained for solid particles.

The comparison of the results of calculations of the air bubbles' relative velocities in water according to the models of Tomiyama, Ishiya, Grace, and Schiller-Neumann with the experimental data (Loth E. 2008) is shown in Fig. 3.

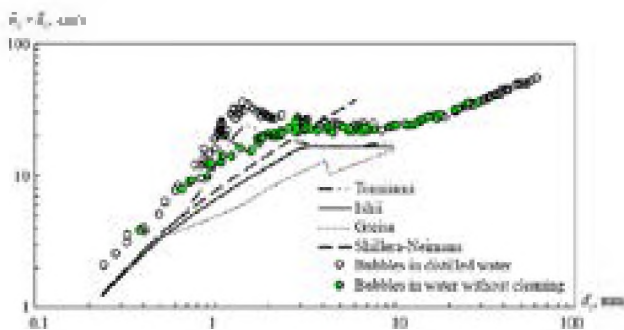


Fig. 3. – Relative velocities of the air bubbles in water.

Due to the fact that the flotation process is carried out in the presence of surfactants in the working liquid, i.e. to determine the coefficient of the bubble resistance, the Ishiya model was chosen:

$$C_D = \max \left\{ \frac{24}{Re} (1 + 0,15 Re^{0,75}); \min \left\{ \frac{2}{3} \sqrt{Eo}; \frac{8}{3} \right\} \right\}, \quad (5)$$

where  $Eo$  – the Eötvös number, showing the relationship between the forces of gravitational and surface tension, which is determined as

$$Eo = \frac{gd_2^2(\rho_1 - \rho_2)}{\sigma}, \quad (6)$$

where  $\sigma$  – the surface tension of the working liquid at the boundary with the gas phase, N/m, determined by the stalagmometric method.

Substituting (4), (5) into (3) and solving together with (1) and (2) as a system of equations with two unknowns, it is possible to determine the maximum allowable relative velocity of the spherical bubble and its diameter.

$$\begin{cases} \frac{18\mu_1(1 + 0,15 \text{Re}^{0,75})(\vec{u}_2 - \vec{u}_1)}{d_2^2} = \vec{g}(\rho_1 - \rho_2); \\ \text{Re} = \frac{\rho_1 d_2 |\vec{u}_2 - \vec{u}_1|}{\mu_1} = 300. \end{cases} \quad (7)$$

To solve the system of equations (7), we will use the following values of physical quantities: the dispersion phase density  $\rho_1=1000 \text{ kg/m}^3$ , the dispersed phase (air) density  $\rho_2=1.2 \text{ kg/m}^3$ , the dynamic liquid viscosity  $\mu_1$ , Pa·s, for water is calculated by the formula

$$\mu_1 = 2,414 \cdot 10^{-5} \cdot 10^{\frac{247,8}{t+133,15}}, \quad (8)$$

where  $t$  – the dispersion phase temperature, °C.

Substituting the values of physical quantities in the system of equations (7) and, replacing the vectors with scalars, let's calculate the roots of the system of equations:  $u_2 - u_1 = 0,18 \text{ m/c}$  and  $d_2 = 2,2 \text{ mm}$ . In this case, the air bubbles relaxation time using equation (4) is  $\tau_2 = 2,2 \cdot 10^{-5} \text{ c}$ .

The obtained roots of the system of equations (7) and the accepted assumptions are consistent with the experimental data. This is evident from Fig. 3: the dependence of the relative air bubble floating up velocities in raw on its diameter in the region of the obtained roots changes its character. This is due to the change in the mode of motion of the bubbles in the liquid.

When a significant amount of the gas bubbles floats in the working medium, a counter-flow of liquid occurs, which tends to replace the volume of the rising gas. Due to the induced counter flow of the liquid, the resistance force of the medium increases.

In the case of motion of ensembles with a large number of particles, the realization of exact solutions in a multiply-connected region turns out to be practically impossible. A detailed comparative analysis of various variants of approximate models was performed in (Warsito W. et.al, 2005), where solutions were obtained for each of them. In the same place (Warsito W. et.al, 2005), the steady-state velocities obtained on the basis of approximate models were compared with numerous experimental data. The most accurate result is the model:

$$\vec{\omega} = -C_{\sigma}(\vec{u}_2 - \vec{u}_1) \left( \frac{\varphi^{\frac{1}{3}} - 1}{\varphi^{\frac{2}{3}} - 1} - \frac{\varphi^{\frac{5}{3}} - 1}{W \left( \varphi^{\frac{2}{3}} - 1 \right)} - \frac{W}{Y} \right), \quad (9)$$

where  $\bar{\omega}$  – the relative velocity of the constrained floating up of the dispersed phase, m/s;

$C_G$  – the coefficient of the constrained velocity, determined by the empirical formula or constant (usually 2/3 – for the particle, 1 – for the bubble);

$\varphi$  – the gas content – the relative volume fraction of the dispersed phase above the aerator,  $m^3/m^3$ ;

W and Y – the parameters that are determined by the formulas (10), (11):

$$W = 3 + \frac{2\mu_1}{\mu_2} + \frac{2}{\pi} \varphi^{\frac{5}{3}} \left( \frac{\mu_2}{\mu_1} - 1 \right); \tag{10}$$

$$Y = 2 + \frac{2\mu_1}{\mu_2} + \varphi^{\frac{5}{3}} \left( 3 - \frac{2\mu_1}{\mu_2} \right); \tag{11}$$

where  $\mu_2$  – the dispersed phase viscosity, Pa·s. For solid particle in the liquid or gas  $\mu_2/\mu_1 \rightarrow \infty$ . Then  $Y \rightarrow 2$ , and  $W \rightarrow 3$ .

According to this model, a graph of dependence of the relative velocities of the dispersed phases' constrained and free motion on the gas content is plotted  $\bar{\omega} / (\bar{u}_2 - \bar{u}_1) = f(\varphi)$  – Fig. 4.

Thus, increase in the volume fraction of the dispersed phase leads to decrease in its relative velocity. In the constrained motion the Reynolds number will also decrease with increase in the volume fraction of the dispersed phase. Therefore, the condition (1) will also be satisfied when the bubble diameter is somewhat larger than the previously calculated. The exact diameter is calculated from the gas content  $\varphi$  whose optimal value is determined experimentally.

There are many different pneumatic aeration devices for flotators. The simplest to manufacture design of the aerator is the aerator made of a perforated tube bent in the form of the Archimedes spiral in a plane parallel to the liquid surface (Fig. 5).

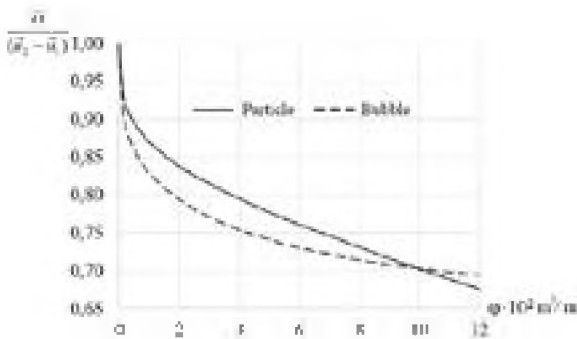


Fig. 4. – Dependence of the ratio of the relative velocities of the dispersed phases' constrained and free motion on their volume fraction

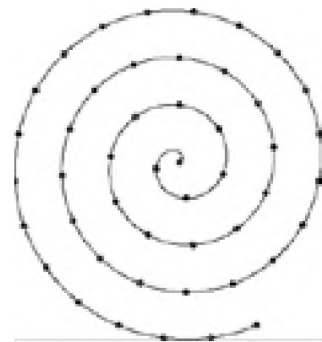


Fig. 5. – the Archimedes spiral

The equation of this curve in polar coordinates ( $r, \alpha$ ) has the form

$$r = \frac{a}{2\pi} \alpha, \quad (12)$$

where  $a$  – the spiral pitch, m.

For the uniform distribution of the gas bubbles over the apparatus' cross section, the aerator's perforation pitch is assumed to be equal to the spiral pitch  $a$ . In this case, each hole (except the boundary) has at least four points in its environment, the distance to which is from  $a$  to  $1.12a$ .

It is quite close to the aerators, made in the form of a tube sheet, in which the perforation is located in the grid nodes with square cells of a size.

The exact solution describing the stages of the bubble growth is not currently obtained due to numerous factors that are not analytically and experimentally definable. So, for example, it is still not possible to accurately take into account the effect of the bubbles on each other as they form in the adjacent holes.

Dependencies connecting the separated bubble diameter  $d_2$  and the aerator's drowned hole diameter  $D_A$  are known for various gas outflow modes (Kantraci M et.al, 2005).

Leibson (Kantraci M et.al, 2005. – for the Reynolds numbers less than 2000. Mu-Young and Blanch (Kantraci M et.al, 2005). – for a wider range of the Reynolds numbers. Kumar and Kulur (Kantraci M et.al, 2005) is the simplest relationship for a wide range of the Reynolds numbers.

Another variant uses the forces equilibrium condition at the moment of the bubble separation to determine its diameter. As the bubble grows, the sticking force and the resistance force compensate for the lifting force due to the difference in the densities between the liquid and the gas. The forces of inertia arising during the bubble growth are usually neglected (Kantraci M et.al, 2005).

When the air bubbles form in the water, there is a difference between the work spent on changing the size of the phase boundary (to overcome the surface tension forces),  $L_\sigma$  and the work spent to overcome the hydraulic resistance,  $L_s$ . Let's write their ratio as

$$\frac{dL_\sigma}{dL_s} = \frac{256\sigma R^3}{9C_{D\rho_1\omega_A^2 R_A^4}} dR \quad (13)$$

where  $R$  – the gas bubble radius, m;

$\omega_A$  – the gas flow velocity in the aerator's hole, m/s;

$R_A$  – the aerator's hole radius, m.

Upon detailed consideration, it can be concluded that when the air bubbles form in the water, the work of changing the size of the phase boundary  $L_\sigma$  is two orders of magnitude more than the work spent to overcome the hydraulic resistance,  $L_s$  already at  $Re > 25$  (for air in the aerator's hole).

Consequently, the forces of hydraulic resistance to the bubble growth can be neglected and the balance of forces at the bubble separation can be written by the formula:

$$\frac{1}{6} \pi d_2^3 g (\rho_1 - \rho_2) = \pi D_A \sigma \sin \theta, \quad (14)$$

where  $\theta$  – the limiting wetting angle, measured by the sessile drop method, degree.

Usually, when calculating the sticking force of the bubble (the right-hand side of the last equation), the wetting angle  $\theta$  is assumed to be  $90^\circ$  [20]. The formula (14) connects the aerator's hole diameter  $D_A$  and the separated bubble diameter  $d_2$  formed on this hole (Kantraci M. et.al, 2005).

To ensure free deposition of non-float particles below the aerator's level, to get them into the sediment, let's assume that the gap between the adjacent aerator tubes with  $D_T$  diameter, m, is equal to three times the maximum linear size of the particles to be separated  $d_p$ , m. Then, the Archimedes' spiral pitch (and the perforation pitch) is

$$a = 3 \cdot d_p + D_T \quad (15)$$

Then the number of the aerator's holes  $N$  can be determined geometrically:

$$N = \frac{\pi D^2}{4a^2} \quad (16)$$

where  $D$  – the internal column diameter, m.

The aerator's tube length  $l$ , m, quite accurately can be determined by the formula

$$l = \frac{D}{2} \sqrt{\pi N}. \quad (17)$$

The gas consumption in the apparatus  $G$ ,  $m^3/s$ , can be expressed in terms of the bubbles motion velocity in the apparatus with the diameter  $D$ , m, and the gas content  $\varphi$  – the ratio of the dispersed and dispersion phases' volumes over the aerator. To do this, let's write the expression for  $\varphi$

$$\varphi = \frac{V_2}{V_\Sigma - V_2}, \quad (18)$$

where  $V_2$  – the gas phase volume in the apparatus,  $m^3$ ;  $V_\Sigma$  – the aerated layer volume,  $m^3$ , determined by the formula

$$V_{\Sigma} = H \frac{\pi D^2}{4}, \quad (19)$$

where H – the height of the liquid's aerated layer, m.

The gas phase volume above the aerator depends on the gas consumption and the bubbles' floating up time  $\tau_L$ , s, which can be expressed through the aerated layer height and the relative velocity of the gas bubbles' constrained motion  $\omega$ , m/s:

$$V_2 = G\tau_L = G \frac{H}{\omega}, \quad (20)$$

where H – the height of the liquid's aerated layer, m.

Let's substitute the volume of the aerated layer, expressed in terms of D and H, together with (20) into (18) and let's express the gas consumption:

$$G = \frac{\pi D^2 \omega \varphi}{4(1 + \varphi)}. \quad (21)$$

The literature (Rubinstein J.B. 1995) recommends that the optimal value  $\varphi$  for the flotation process should not exceed 5%, since a further increase may lead to strong mixing of the working liquid due to the induced counter flow of the liquid during the dispersed phase motion in a limited space (due to the volumes' replacement).

Using the proposed formulas for calculating the aerator's design, the pneumatic aerator's parameters for the apparatus with a wide interval of the column diameter can be calculated.

**Comparison of the results of theoretical studies with experimental data.** To verify the accepted assumptions and the adequacy of the proposed mathematical expressions, the pneumatic aerator's design and technological parameters were calculated. Based on the calculated parameters, the laboratory installation of the column flotator with the pneumatic aeration of the liquid was created. Its image is shown in Fig. 6.



Fig. 6. – The laboratory installation of the column flotator with the pneumatic aeration of the liquid

The values of the created laboratory flotator's main parameters are the following: the column's cylindrical part height is 1 m; its internal diameter is 0.2 m; the aerator's tube length is 1.6 m; its outside diameter is 6 mm; the aerator's holes number is 80 pieces; their diameter is 0.3 mm; the spiral pitch and the perforation pitch are 0.02 m.

The bubbles dispersion formed at the liquid aeration was studied in the created installation. Their size was measured by the method of photographing the air bubbles in the volume of liquid and on its surface together with the scale ruler as is shown in Fig. 7.

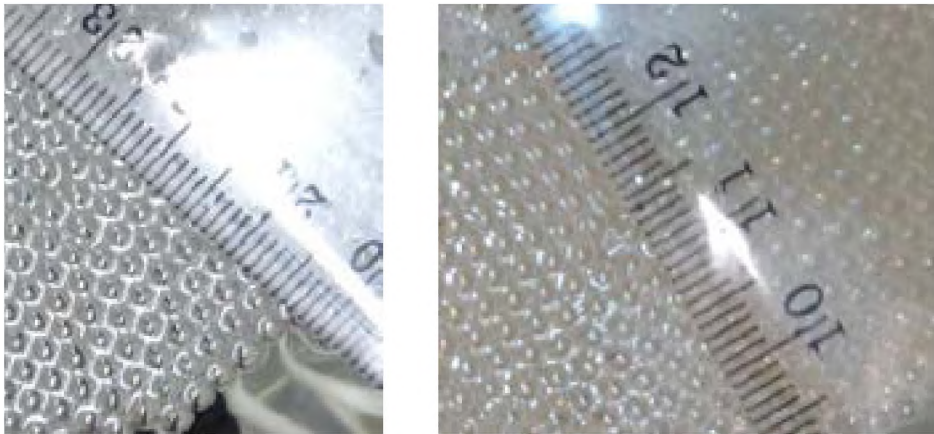


Fig. 7. – The bubbles dispersion determination on the liquid surface

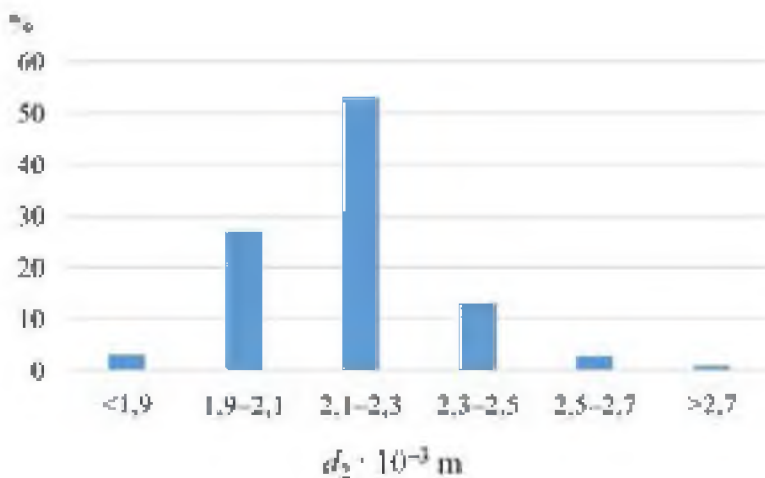


Fig. 8. – The bubbles distribution by sizes

Comparative analysis of the experimental and calculated values of the average bubble diameter.

The value of the experimental average bubble diameter ( $2.3 \cdot 10^{-3}$  m) was compared with the result of the gas bubbles calculations formed at the drowned hole. The high convergence of the calculated and experimental data allows to speak about the adequacy of the used mathematical dependencies and their successful application in the process of creating the flotator for separating the mixture of ground materials.

Using the condition of equality of the densities of the liquid and the “bubble – particle” complex, it is possible to determine the boundary equivalent diameter of the floating particles. The “bubble – particle” complex formed during the flotation with the equivalent particle diameter smaller than the boundary one will float into the foam layer. For the bubble diameter, which was determined using the system of equations (7) –  $d_2 = 2,2 \cdot 10^{-3}$  m, with the particles density  $1100 \text{ kg/m}^3$  their boundary equivalent diameter will be  $4.7 \cdot 10^{-3}$  m, and with the particles density  $2500 \text{ kg/m}^3$  –  $1.9 \cdot 10^{-3}$  m.

**Conclusions.** The features of the physicochemical properties of plastics make it necessary to take them into account when creating new equipment for each stage of the flotation separation technology of the mixture of ground plastics.

Taking into account these features, the paper proposed mathematical dependencies taking into account the condition of the spherical gas bubbles preservation, their interaction with the surrounding dispersion medium and with each other, the condition of floating up of the “bubble-particle” complex and the ratio of the dispersed phases’ volumes in the complex. The mathematical dependencies with known properties of the floating material allow to assess the feasibility of carrying out the flotation separation process of the mixture of ground plastics in the column apparatus with the pneumatic aeration and determine the design and technological parameters of the aerator (number of the aerator holes, their diameter, the spiral pitch and required tube length).

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