Multi-layer cross-country maps as a data source for solving the problem of laying a route

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Abstract. Representation of information about the surface of movement in the form of discrete maps of patency is used in problems related to the management of autonomous vehicles. The technology was developed for use on rovers. The areas of autonomous transport, transport and technological machines operating in an artificial and natural environment, mobile robots seem to be promising. The principle of building maps of patency and methods for choosing the trajectory of movement have a great potential for expanding the volume of information entered into the cells of the map, variability of the approach to the criteria for optimality of the trajectory, etc. The basics of the technology for preparing information about the area, the principles of using this information in solving the transport problem are presented, simple examples are given.

1 Introduction

The actual state of the environment imposes a number of restrictions on the mobility of the chassis. Mobility, as a more general parameter, includes patency, stability and controllability.

The cross-country ability is traditionally divided into geometric (dimensional restrictions) and support-coupling (restrictions on the transmission of traction force). Stability and controllability are maintained by the spherical robot in a relatively small range of angles of inclination of the support surface.

Thus, the area under consideration can be divided into zones that are impassable for a car with specified parameters and zones in which movement is allowed. The maximum size of the zone is represented by a cell in the form of a square with a side equal to the largest overall size of the chassis (usually this is the overall length of the vehicle). The set of such cells is combined into a discrete working field. Each cell is associated with a certain value that describes the possibility of overcoming the cell. In the simplest case, for example, "0 "if the

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cell is allowed to move, or" 1 " if the cell contains an insurmountable obstacle. The described discrete working field is known as the "passability map" (see, for example, publications [1]).

The principles of filling surfaces with geometric primitives known in topology and the variability of the direction of movement of the machine through the cells of a discrete working field lead to the assumption that it is advisable to use a hexagonal shape of cells, and each hexagon is built on the basis of an inscribed circle with a diameter equal to the largest overall size of the chassis.

In this article, we will limit ourselves to working with cells in the form of a square for the sake of simplifying the graphical interpretation of the problem.

Obviously, for cells that are allowed to move, it would be useful to associate with each cell a maximum of data that is useful for solving the problems being solved. The form of representation from a mathematical point of view is a vector, each component of which is responsible for its own parameter. Thus, we get the concept of a multi-layer patency map, containing, for example, a patency layer, a layer of energy consumption [1], etc.

For example, to assess the energy consumption on the route being formed, it is advisable to divide the area under consideration into zones conditionally limited by isolines – lines of identical energy consumption. In this generalized form, we can talk about the "energy consumption layer".

To plot the trajectory, it is advisable to represent the terrain in the form of a discrete working field. The cells of the working field are square, the side of the square is equal to the diameter of the sphere. Each cell has properties. We describe these properties in the form of a matrix of relative energy consumption, setting the relative energy consumption for impassable zones as "excessive" for the chassis under consideration (for example, ∞).

The purpose of the study is to develop the principles of constructing and using multi-layer cross-country maps for mobile chassis.

The objects of research are transport and transport-technological wheeled and tracked vehicles.

Research methods – The basic method is the representation of the terrain through which the mobile platform moves, in the form of a set of cells of a discrete working field. Each cell has a characteristic size determined by the largest size of the chassis under consideration. Each cell is characterized by a set of data represented as a vector. Each component of the vector is thus part of a certain surface, which can be called a layer. Thus, the map contains layers of passability, energy efficiency, etc. The route can be laid according to the algorithms of a group of methods related to the Lee method, based on the specifics of the formulation of the transport problem

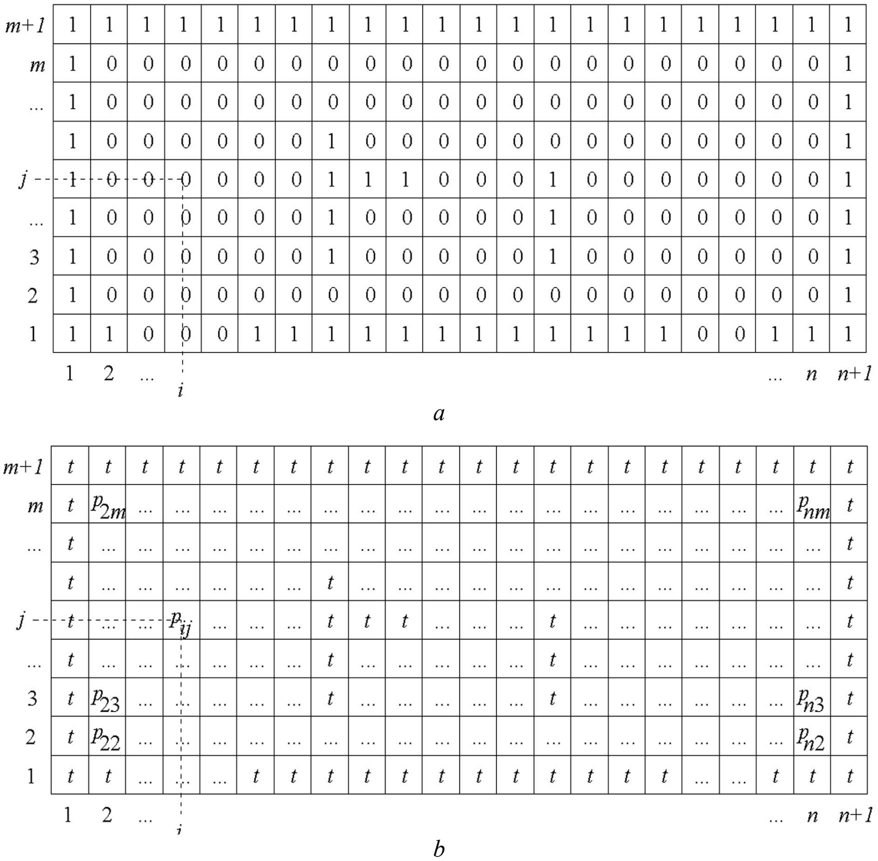
2 Problem statement

Working with literary sources and analyzing the experience of designing the chassis of transport and traction machines [2, 3, 4] allow us to formulate the following tasks.

1. Based on the analysis of the industry experience, we propose an approach to the development of the concept of discrete traffic maps.
2. To propose a mathematical apparatus and a methodology (sequence of actions) for laying a route using multi-layer cross-country maps.
3. To test the model on the example of constructing a trajectory on a local discrete working field containing information about obstacles.

3 The results of theoretical research

To automate the calculated estimation of energy consumption on the trajectory, it is proposed to apply an energy consumption layer containing information formed on the basis of calculations or obtained experimentally. The energy consumption layer, as well as the patency layer itself, is based on a discrete working field of size $(n+1)\times(m+1)$ with square cells with a side length equal to the maximum size of the chassis (Figure 1, *a*).



$m+1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
...	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
j	1	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	1
...	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1
3	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
	1	2	...	i																...	n	$n+1$

a

$m+1$	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
m	t	p_{2m}	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	p_{nm}	t
...	t	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	t
	t	...	...	...	...	...	...	t	...	...	...	...	...	...	...	...	...	...	...	...	...	t
j	t	...	...	p_{ij}	...	...	...	t	t	t	...	...	...	t	...	...	...	...	...	...	...	t
...	t	...	...	...	...	...	...	t	...	...	...	...	...	t	...	...	...	...	...	...	...	t
3	t	p_{23}	...	...	...	...	...	t	...	...	...	...	...	t	...	...	...	...	...	...	p_{n3}	t
2	t	p_{22}	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	p_{n2}	t
1	t	t	...	...	...	t	t	t	t	t	t	t	t	t	t	t	t	...	...	t	t	t
	1	2	...	i																...	n	$n+1$

b

Fig. 1. Geometric interpretation of a discrete working field in the form of maps: *a* – passability; *b* – a map of energy consumption (or-a layer of energy consumption as part of a multilayer passability map).

Unlike the cells of the patency layer, each cell of the energy consumption layer (for the sake of further formalization of the problem, we can talk about sublayers; these sublayers are similar to the layers of the patency map and are a convenient illustration for understanding the principles of using a multilayer map) is characterized by four parameters that make up the vector p_{ij} (Figure 1, *b*).

Each of these parameters characterizes the relative energy consumption when moving through the cell in one of four directions (parallel to the coordinate axes). In the simplest case (in the absence of significant aerodynamic drag of the medium) this is the ratio of the coefficient of resistance to movement for this cell to the coefficient of rolling resistance f_0 of the chassis on a horizontal non-deformable surface. If the cell is impassable for the chassis, the parameter takes a certain value t, which significantly exceeds the energy capabilities of the machine (Figure 2).

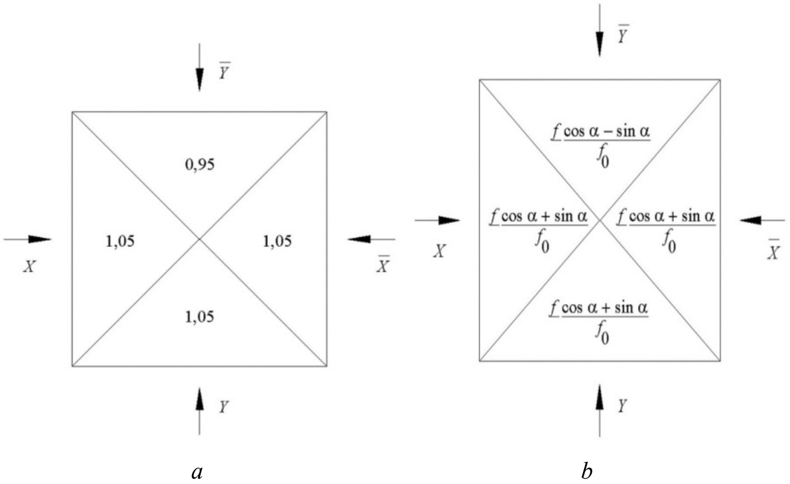


Fig. 2. Graphical representation of the energy consumption map cell: *a* – concretized; *b* – with a symbolic description (*f* and *f*0 are the coefficients of resistance to movement on the terrain and on a non-deformable base, α is the angle ofinclination of the movement surface).

The energy consumption map can be directly used as a basis for laying a route using the same information processing algorithms that were used for the traditional cross-country map. The advantage is to reduce the number of computational operations when evaluating energy consumption on the trajectory.

For the experimental construction of the energy consumption layer, ideally, each cell on a discrete working field should be traversed by the chassis at least once in each of the four directions. For machines with an electric drive, energy consumption can be determined by taking advantage of the fact that the torque developed by the electric motor is directly proportional to the current strength. For a known speed of motion v, it is possible (for uniform motion) to use the dependence (1):

$$\left(\Delta E_{\psi}\right)_{ij} = UI\,D/\left(v\eta_{m}\right) \tag{1}$$

Here U and I are the on-board voltage and current in the traction drive motor circuit, ηm is the efficiency of the gear part of the drive (a constant value). At a constant speed of movement and a constant supply voltage, the relative energy consumption will be equal to the ratio of the currents consumed by the traction drive motor during the passage of this cell and when moving on a non-deformable horizontal surface.

The total energy consumption (at known values of the mechanical efficiency of the gear parts of the robot drives) can be determined by adding up the energy consumption for all the drives used in the motion system.

The problem of finding a trajectory can be reduced to the case of finding the shortest trajectory of movement between two points, subject to specified restrictions, or to finding a route that provides a minimum of energy consumption.

It is also possible to take into account many factors in a comprehensive manner. In this case, they resort to the procedure of constructing a generalized response [1].

Let's consider the methods of constructing the trajectory of the vehicle. In all cases, the coordinates of the start and finish points are known, and the properties of the movement surface are also set.

4 The Lie algorithm

The Lie algorithm refers to algorithms for finding the shortest path on a planar graph (a kind of"breadth-first search"). It is used when tracing printed circuit boards, plotting routes in games, when searching for the path of robot transport vehicles (for example, planetary rovers) [2].

The algorithm works on a discrete working field (a figure bounded by a closed line, in a particular case – a rectangular one), divided into rectangular cells (in a particular case – square).

The set of all cells of a discrete working field is divided into subsets of cells:

- "passable" (free), i.e. when searching for a path, they can be passed,
- "impassable" (obstacles), the path through this cell is prohibited.

The starting cell (source) and the finishing cell (receiver) are highlighted.

In the case of working with a transport (transport-technological) machine for the chassis under consideration, a terrain terrain map is built according to this principle [2].

The algorithm implements the stages:

- initialization;
- wave propagation;
- restore the path.

During initialization, an image of a set of cells of the processed field is constructed, the attributes of the cells are determined – passability / impassability, start / finish.

Wave propagation is carried out by checking the patency of cells adjacent to the starting point. Adjacent cells can be considered as a Moore neighborhood (eight cells, including diagonal ones) or a von Neumann neighborhood (four cells, only orthogonal to the given one. The wave can have the shape of a rectangle/square or a rhombus [2].

If a cell is passable and has not been used before, its ordinal number on the generated trajectory is written to its attribute. This point becomes the starting point and the process of laying the path is repeated.

The trajectory is restored in reverse order. The process starts from the finish point, the next point is selected that has an attribute one less.

Related algorithms are known that are used in routing (in particular, when tracing printed circuit boards):

- Hadlock, "A shortest path algorithm for grid graphs" (algorithm for finding the shortest path for grid graphs) Networks, 1977;
- Soukup (Soukup), "Fast maze router" (Fast routing in the maze) DAC-78;
- Mikami & Tabuchi, "A computer program for optimal routing of printed circuit connectors", IFIP, H47, 1968;
- Hightower, "A solution to the line-routing problem on the continuous plane", DAC-69.

All these algorithms, however, do not guarantee finding the shortest path. There may be several trails with the same length and additional conditions (criteria) for selecting the best one are needed. In addition, formally, the Lie algorithm works in such a way that a trajectory consisting of orthogonal segments is constructed. The trajectory straightening requires modification of the algorithm.

To speed up the algorithm, it is advisable to consider the current start and finish points as wave sources [2].

5 The principle of a complete search of trajectory

The principle of a complete search of trajectory options differs from the Lie algorithm by the principle of considering the neighborhoods of the starting point. The algorithm is suitable for working with groups of control objects and includes the following steps:

1. The optimization task is set: the entire group must move from a set of start points $\{A\}$ to a set of finish points $\{B\}$ with a minimum of time and energy consumption.
2. A layer of energy consumption is built in a multi-layer cross-country map. The problem becomes discrete, since it is solved on the set of cells into which the map is divided.
3. All possible routes from $\{A\}$ to $\{B\}$ are built for each control object. To solve the problem by iterating over the trajectories, you will need to introduce additional rules – set the direction of traversing the branches of the graph of the route option counterclockwise and eliminate the threat of collision of control objects on the routes (for example, by laying a pass on the principle of interference on the right.
4. For each route, the travel time and total energy consumption for movement are calculated.
5. A combination of routes is built (C_k^n the number of options, if there are n routes, k control objects, and the options are not repeated).
6. For each combination, the value of the optimality criterion (also known as the efficiency criterion) is calculated, for example, according to the Harrington method [2,3]. Such values are entered in the vector D.
7. The values from item 5 are compared by iteration and the best option is selected: $d_{opt} = \left\| \mathbf{D}^* \right\|_\infty$, here $\mathbf{D}^* : \left\{ d_j{}^* \right\} = \left\{ d_j^{-1} \right\}, j = \overline{\left(1, m\right)}$, m is the number of components of the vectors $\mathbf{D}$ and $\mathbf{D}^*$.

The disadvantage of the algorithm is a larger amount of computational operations compared to the Lee algorithm. In addition, the found trajectory will consist of orthogonal segments and requires straightening.

Advantages-the algorithm provides for solving the tracing problem simultaneously for a group of control objects and optimizing the route, in particular, in terms of energy consumption for movement.

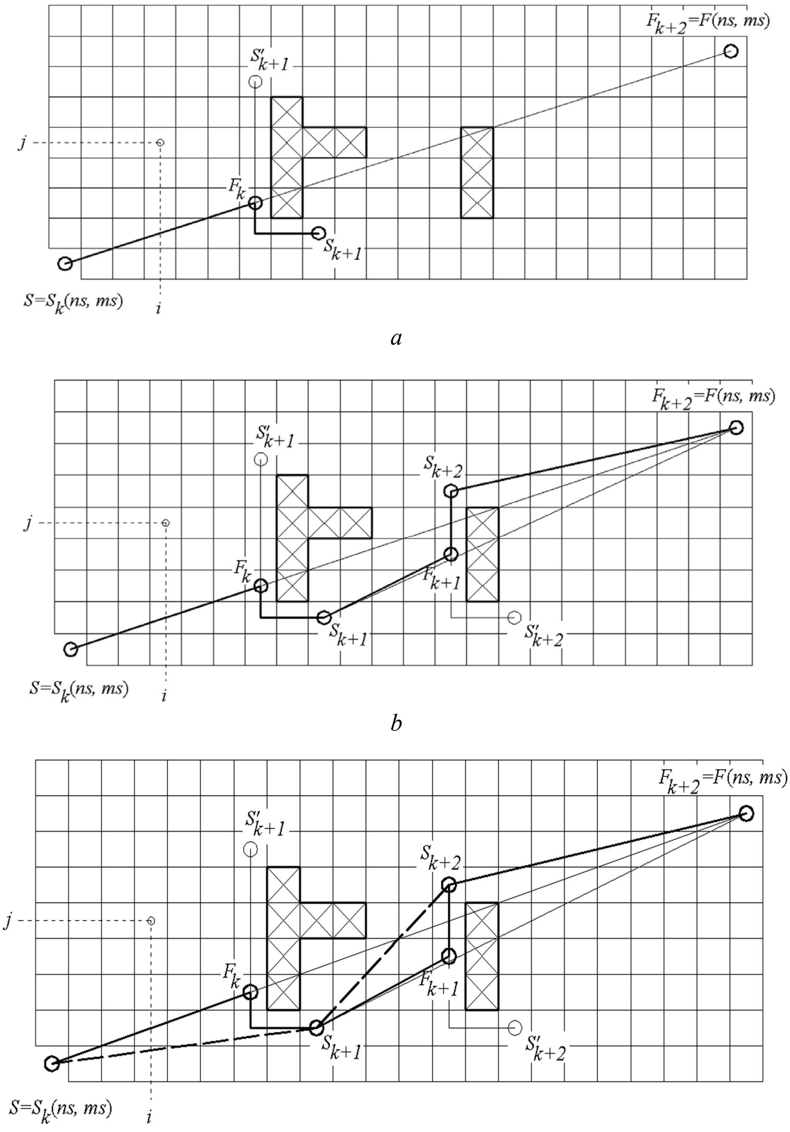
6 The principle of laying a route of the minimum length

The algorithm is also implemented on a discrete working field-a cross-country map. You can use the energy consumption map as a working field. The start and finish points are set before the implementation of the algorithm and lie within the working field.

The algorithm includes the following steps (see also Figure 3):

1. Assigning the current start and finish points.
2. Construction of the shortest trajectory (straight line segment) between these points.
3. Construction and discretization of the traffic corridor of the control object.
4. Identification of the first obstacle.
5. Construction of two options for the trajectory of the obstacle circumference, the choice of the shortest path.
6. Checking the feasibility of straightening the trajectory. Straightening the trajectory.
7. Reassign the start point and go to step 1.

The result of the algorithm is a trajectory, which is a polyline consisting, in general, of non-linear segments, which distinguishes this trajectory from the result of the Lie algorithm. The process ends if the current starting point coincides with the finish point or there is no solution (the finish point is not available).



a

b

c

Fig. 3. Example of laying a route on a cross-country map: *a* – the first steps of the algorithm; *b* – the algorithm is executed, the polyline straightening function is disabled; *c* – the algorithm is executed, the dashed bold polyline shows the optimized section of the trajectory.

7 The principle of estimating energy costs when laying a route

The principle of estimating energy costs when laying a route is reduced to summing up energy costs on the considered trajectory for the cells of a discrete workspace along which the robot chassis movement corridor passes.

8 The construction of a lane (corridor)

The construction of a lane (corridor) requires known current values of the position of the start and finish points with known coordinates: S (ns, ms) and $F(nf, mf)$. Here ns, ms, nf, $mf \in N$ are the coordinates of the cell centers.

The equation of the straight line: $y = ax + b$.

The coefficients of the equation of the line passing through the points $S(ns, ms)$ and $F(nf, mf)$ are determined by the expressions (2):

$$a = (ms - mf)/(ns - nf) \text{ and } b = ms - a \cdot ns \tag{2}$$

The width of the corridor is set, let's assume it is equal D.

The corridor is bounded by two straight lines defined by the system of equations (3):

$$\begin{cases} y_1 = a_1 \cdot x + b_1 \\ y_2 = a_2 \cdot x + b_2 \end{cases}. \tag{3}$$

The offset of the lines bounding the corridor relative to the corridor axis, deferred along the ordinate axis, is denoted by y_D.

Since the coefficient $a = tg\ \alpha$, where α is the angle of inclination of the straight line (the axis of the trajectory), the desired displacement of the straight lines relative to the trajectory line is determined by the expression (4):

$$y_D = \frac{0,5D}{\cos\ \alpha} = \frac{0,5D}{\cos\left(\text{arctg}\ \dfrac{ms - mf}{ns - nf}\right)}. \tag{4}$$

Expression (4) is applicable when constructing a corridor for an arbitrary shape trajectory – the lines bounding the corridor will be equidistant.

Then $ms_1 = ms + y_D$ and $ms_2 = ms - y_D$; $mf_1 = mf + y_D$ and $mf_2 = mf - y_D$;

Accordingly: $a_1 = (ms_1 - mf_1)/(ns - nf)$; $b_1 = ms_1 - a \cdot ns$; $a_2 = (ms_2 - mf_2)/(ns - nf)$ and $b_2 = ms_2 - a \cdot ns$.

To ensure the passage of the control object along the corridor, it is necessary that all the cells through which the corridor is laid are suitable for the passage of the machine.

Therefore, the next step is to perform sampling of the corridor (Figure 4).

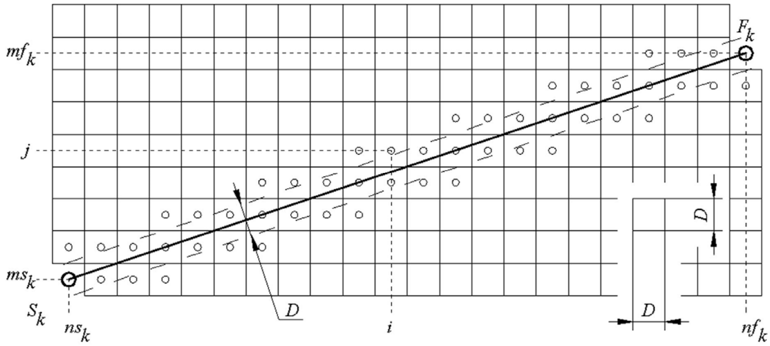


Fig. 4. Sampling of the corridor.

In Fig. 4, the corridor is built between the points S_k (ns_k, ms_k) and F_k (nf_k, mf_k).

The symbols indicate the cells affected by the corridor.

The cell with the coordinates of the center (i, j) is bounded by straight lines:

$$\begin{cases} y = j + 0.5 \\ y = j - 0.5 \\ x = i + 0.5 \\ x = i - 0.5 \end{cases}. \tag{5}$$

To identify the intersection with the boundaries of the strip, we will set the cell number $i = \overline{1, nf_k}$

If one of the conditions is met

$$\begin{cases} j - 0.5 \le y_1(i - 0.5) \le j + 0.5 \\ j - 0.5 \le y_2(i - 0.5) \le j + 0.5 \\ j - 0.5 \le y_1(i + 0.5) \le j + 0.5 \\ j - 0.5 \le y_2(i + 0.5) \le j + 0.5 \end{cases}, \tag{6}$$

this means that the cell (i, j) is affected by the center coordinates of the corridor boundaries.

The number of such a cell (for example, its coordinates) is added to the list of cell numbers involved in the formation of the corridor.

Thus, if **K** is the set of map cells that make up the SR movement corridor, then

$$\begin{gathered} (i, j): [(j - 0.5) \le y_1(i - 0.5) \le (j + 0.5)] \cup [(j - 0.5) \le y_2(i - 0.5) \le (j + 0.5)] \cup \\ \cup [(j - 0.5) \le y_1(i + 0.5) \le (j + 0.5)] \cup [(j - 0.5) \le y_2(i + 0.5) \le (j + 0.5)] \in \mathbf{K} \end{gathered} \tag{7}$$

9 The bypass of the obstacle

The bypass of the obstacle is estimated in two directions (see Figure 3). The trajectory is laid along the cells marked as free along the contour of the obstacle. To optimize the trajectory, the possibility of straightening it between the inflection points is considered.

The condition for the end of the bypass process is the possibility of building a corridor to the finish point, in which it is possible to move forward by at least one cell.

The obtained trajectories are compared in length, and the shortest one is selected.

10 Practical consequences and prospects

It seems advisable to use multi-layer cross-country maps when solving problems of controlling single mobile machines and groups of transport or transport-technological machines, including those working on the surfaces of other planets.

The principles described in the article are applicable to solving the problem of partial automation of traffic control (active assistance to the operator, automation of routine tasks related to navigation, etc.).

With regard to the organization of management of autonomous machines of the agricultural complex [3,4,5,6,7], multi-layer cross-country maps are the simplest and most promising basis for ensuring the operation of such machines with minimal investment.

11 Conclusions

The use of multi-layer cross-country maps will allow automating the work on laying the trajectory, solving logistics problems in relation to individual mobile chassis and groups of transport and transport-technological machines.

It becomes possible to optimize the route according to various criteria, which opens the way to the construction of adaptive algorithms for managing individual objects and the transport system as a whole.

It can be expected that the use of the proposed approach will reduce the total energy consumption for performing transport and other operations against the background of improving the safety of machine operation.

The work is carried out in accordance with the Strategic Academic Leadership Program "Priority 2030" of the Kazan Federal University of the Government of the Russian Federation.

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